C.U.SHAH UNIVERSITY
WinterExamination-2015

Subject Name:Mathematics-II

Subject Code: 4SC02MTC1   **Branch: B.Sc. (All)**

Semester: II   **Date: 21/11/2015**   **Time: 10:30 To 01:30**   **Marks: 70**

Instructions:
  (1) Use of Programmable calculator & any other electronic instrument is prohibited.
  (2) Instructions written on main answer book are strictly to be obeyed.
  (3) Draw neat diagrams and figures (if necessary) at right places.
  (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**

a) If $x + iy = \sqrt{2} + 3i$, then $x^2 + y$ is

  1) 7   2) 5

  3) 13   4) $\sqrt{2} + 3$

b) Solve: $y'' - 3y' + 2y = 0$.

  1) $c_1 e^{-x} + c_2 e^{2x}$ 2) $c_1 e^{x} + c_2 e^{2x}$

  3) $c_1 e^{x} + c_2 e^{-2x}$ 4) None of these

c) If $f(z) = e^{2z}$, then the imaginary part of $f(z)$ is

  1) $e^{y} \sin x$ 2) $e^{x} \cos y$

  3) $e^{2x} \cos 2y$ 4) $e^{2x} \sin 2y$

d) Hyperbolic functions are periodic. Determine whether the statement is true or false.

e) $\sin ix = -i \sinh x$. Determine whether the statement is true or false.

f) $\dfrac{e^{x} + e^{-x}}{2} = $ _____.

  1) $\cosh x$ 2) $\sinh x$

  3) $\cos x$ 4) None of these

g) The particular integral of $(D^2 - 4)y = \sin 3x$ is

  1) $\dfrac{1}{4}$ 2) $-\dfrac{1}{13}$

3) $\dfrac{1}{5}$

4) None of these

h) $\int_0^{\frac{\pi}{2}} \sin^6 x$ is equal to

1) $\dfrac{16}{5}\pi$

2) $\dfrac{5}{16}\pi^2$

3) $\dfrac{5}{16}\pi$

4) $\dfrac{5}{16}$

i) $\dfrac{1}{D^2}x^2 = $ _____.

j) If the root of $f(D) = 0$ are $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$ then the complementary function of $f(D)y = X$ is

1) $c_1 e^{\alpha x} + c_2 e^{\beta x}$

2) $\alpha e^{m_1 x} + \beta e^{m_2 x}$

3) $e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

4) None of these

k) S is a sequence if

1) $S: R \to N$ is a function

2) $S: N \to R$ is a function

3) $S: R \to R$ is a function

4) None of these

l) Number of arbitrary constants in the equation of a cone is :

1) 3

2) 4

3) 5

4) 7

m) The equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{2z}{c}$ represent :

1) an ellipsoid

2) a hyperboloid

3) an elliptic hyperboloid

4) a hyperbolic paraboloid.

n) All the generator of a cylinder meet at a point. Determine whether the statement is true or false.

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	a)	State and prove De Moivre's Theorem.	**(05)**

b) Prove that $\sin^{-1} z = \dfrac{1}{i}\log(iz \pm \sqrt{1 - z^2})$. **(05)**

c) Expand $\sin^8 \theta$ in terms of cosine series of multiple angles of θ. **(04)**

Q-3   **Attempt all questions**   **(14)**

a)   Solve: $x^5 + 1 = 0$.   **(05)**

b)   Write formula of $\log(x + iy)$ and find $\log\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$.   **(05)**

c)   Prove that $\left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n = \cos(n\theta) + i\sin(n\theta)$.   **(04)**

Q-4   **Attempt all questions**   **(14)**

a)   Solve: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = e^{3x}$.   **(05)**

b)   Solve: $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$ **(05)**

c)   Solve: $y''' - 7y' - 6y = 0$.   **(04)**

Q-5   **Attempt all questions**   **(14)**

a)   Solve: $x^3\frac{d^3y}{dx^3} - x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = x^3 + 3x$.   **(05)**

b)   Prove that :   **(05)**
    $i)\ \sin(x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y$
    $ii)\ \cosh^2 x - \sinh^2 x = 1$.

c)   Solve: $y'' + 4y = \sin 2x$ **(04)**

Q-6   **Attempt all questions**   **(14)**

a)   Prove that $\int \cos^n x\ dx = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{(n-1)}{n} \int \cos^{n-2} x\ dx$.   **(05)**

b)   Evaluate: $i)\ \int_0^{\pi} \sin^5\frac{x}{2}\ dx$, $ii)\ \int_0^{\pi} \sin^6 x \cos^4 x\ dx$.   **(05)**

c)   Every convergent sequence has a unique limits.   **(04)**

Q-7   **Attempt all questions**   **(14)**

a)   Find the equation of cone whose vertex is (α, β, γ) and base curve   **(05)**
    $y^2 = 4ax, z = 0$.

b)   Prove that the equation of a cone whose vertex is the origin is homogeneous and conversely.   **(05)**

c)   Prove that the cones $ax^2 + by^2 + cz^2 = 0$ and $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$ are reciprocal.   **(04)**

Q-8   **Attempt all questions**   **(14)**

a) Find the equation of right circular cylinder of which guiding curve is, a circle $x^2 + y^2 + z^2 = 4, x + y + z = 3$. **(05)**

b) Write equation and draw rough sketch of at least five different conicoids. **(05)**

c) Find the equation of a cylinder whose generating lines have the direction cosine (l, m, n) and which passes through the circle $x^2 + z^2 = a^2, y = 0$. **(04)**